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A SYSTEM OF CONCENTRATED AND DISTRIBUTED HEAT SOURCES OVER A TRANSLATIONALLY AND ROTATIONALLY MOVING CYLINDER*

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Abstract—This paper presents an analysis of the heat conduction in a translationally and rotationally moving cylinder under the influence of a system of concentrated and distributed heat sources over the cylindrical surface. Steady, transient and unsteady states are considered. Physically, this problem represents the machining of a cylinder by a set of tools. Two simpler cases are also investigated: A system of moving ring sources and a moving generatrix-band source.

NOMENCLATURE

a_0, a_n, b_n, constants defined by (25);
B_{nm}, function defined by (24);
c, specific heat;
g, Green's function for transient state;
G, Green's function or fundamental Green's function;
h, convective heat-transfer coefficient;
k, thermal conductivity;
l, half-width of the cutting tool surface;
L, distance between centers of tools;
N, number of tools;
Q, rate of heat generation;
Q', rate of heat generation per unit length;
Q'', rate of heat generation per unit area;
r, θ, x, cylindrical coordinates;
r_0, radius of the cylinder;
t, time;
T, temperature;
u, axial velocity;
U, V, functions defined by (46).

Greek symbols

α, thermal diffusivity;
δ, Dirac delta function;
ν, angular half-width of the cutting tool surface;
ρ, density;
ω, angular velocity;
ω_n, $= \sqrt{(\omega n/\alpha)}$;
ζ_1, ζ_2, functions defined by (48).

INTRODUCTION

THE THEORY of moving heat sources is of great importance in manufacturing and metallurgical processes and has been under intensive study for almost seven decades [1–5]. Most of the analyses are for systems with a concentrated source moving along a straight coordinate.

Jaeger [6] was probably the first to investigate the heat conduction in a circular cylinder around whose surface a line source moves with constant angular velocity. He applied Laplace transform to the heat equation and solved the transformed equation by the classical method [7] with the aid of convolution theorem. According to Jaeger, the advantage of his method is to separate the more interesting steady or periodic solution. Most recently, DesRuisseaux and Zerkle [8] applied Jaeger's method to the study of an infinite band source with convective heat transfer at the cylinder surface.

The usual method for the solution of a moving heat source problem is first to find the instantaneous source, then to change the stationary coordinate system to a moving one and to integrate with respect to time. This results in the unsteady solution if the upper limit of integration is set to any value of time and in the steady solution if the time limit is extended to infinity [2]. This method, however, usually yields the steady solutions in double and triple series for two and three dimensional problems. It was probably due to this drawback that Jaeger recommended the use of the Laplace transform–convolution method.

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The present study is concerned with the heat conduction in a translationally and rotationally moving cylinder under the influence of a set of concentrated and distributed heat sources. Physically, this problem represents the machining of a cylinder by a set of grinders or cutters. Two simpler cases are also considered: One for moving ring sources and the other for a line source moving over the circumference of a cylinder.

FUNDAMENTAL EQUATIONS

Consider the machining of a long circular cylinder by a set of tools as schematically shown in Fig. 1. The cylinder rotates counterclockwise at a constant angular velocity ω and moves axially at a constant velocity u in

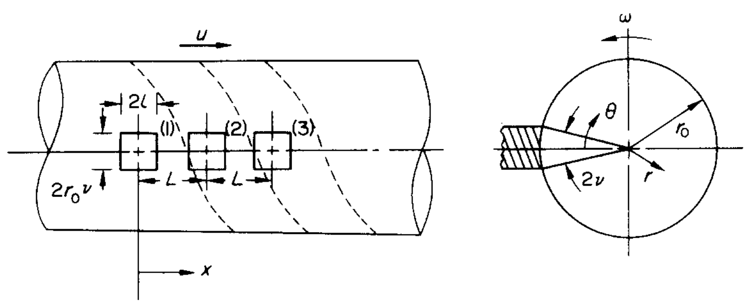

FIG. 1. Geometry of the problem.

the positive x direction. Due to the removal of material from the cylinder surface, heat is generated between the cylinder and the tools over the finite areas:

$$-\nu < \theta < \nu, \quad (pL-l) < (pL+x) < (pL+l)$$

where $p = 0, 1, 2, \ldots, N-1$ with N being the number of tools and other quantities are defined in the Nomenclature. If, as usual, thermo-physical properties are assumed constant, the differential equation governing the heat conduction in unsteady state may be written in the form

$$\frac{1}{\alpha}\left(\frac{\partial T}{\partial t}+\omega\frac{\partial T}{\partial \theta}+u\frac{\partial T}{\partial x}\right)-\nabla^2 T = 0, \tag{1}$$

where ∇^2 is the Laplacian operator

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}+\frac{\partial^2}{\partial x^2}, \tag{2}$$

for $0 \leqslant r < r_0$, $0 \leqslant \theta \leqslant 2\pi$, $|x| < \infty$. The boundary conditions on T may be written as follows:

$$T = F(r,\theta,x) \quad \text{for} \quad 0 < r < r_0, \quad 0 \leqslant \theta \leqslant 2\pi, \quad |x| < \infty, \quad t = 0; \tag{3}$$

$$k\frac{\partial T}{\partial r} + hT = f(\theta,x) \quad \text{for} \quad r = r_0, \quad 0 \leqslant \theta \leqslant 2\pi, \quad |x| < \infty, \quad t > 0; \tag{4}$$

$$T = 0 \quad \text{for} \quad 0 < r < r_0, \quad 0 \leqslant \theta \leqslant 2\pi, \quad |x| = \infty, \quad t > 0; \tag{5}$$

$$k\frac{\partial T}{\partial r} = Q''(r_0,\theta,x) \quad \text{for} \quad r \to r_0, \quad -\nu < \theta < \nu, \quad (pL-l) < (pL+x) < (pL+l), \quad t > 0. \tag{6}$$

If heat is generated in N points instead of N finite areas, condition (6) is replaced by

$$k\frac{\partial T}{\partial r} = \frac{Q}{4\pi R^2} \tag{6'}$$

for $R \to 0$, $\theta = 0$, $x = pL$, $t > 0$ where $R^2 = r^2+r_0^2-2rr_0\cos\theta+x^2$. The known functions f, F, Q, and Q'' are assumed as square integrable with respect to space variables. This condition is usually satisfied in practical problems.

To find the solution of (1) satisfying conditions (3) through (6), it is convenient to find first the corresponding Green's function which is to satisfy the following equations:

$$\frac{\partial G}{\partial t}+\omega\frac{\partial G}{\partial \theta}+u\frac{\partial G}{\partial x}-\alpha\nabla^2 G=\frac{1}{r}\delta(r-r')\delta(\theta-\theta')\delta(x-x') \tag{7*}$$

for $0<r<r_0, 0\leqslant\theta\leqslant 2\pi, |x|<\infty, t>0$;

$$k\frac{\partial G}{\partial r}+hG=0 \quad \text{for} \quad r=r_0, \quad 0\leqslant\theta\leqslant 2\pi, \quad |x|<\infty, \quad t>0 \tag{8}$$

$$G=0 \quad \text{for} \quad 0\leqslant r<r_0, \quad 0\leqslant\theta\leqslant 2\pi, \quad |x|<\infty, \quad t=0 \tag{9}$$

$$G=0 \quad \text{for} \quad 0\leqslant r<r_0, \quad 0\leqslant\theta\leqslant 2\pi, \quad |x|=\infty, \quad t>0. \tag{10}$$

It is to be noted that $G(r,\theta,x,t|r',\theta',x')$ is a continuous point source instead of an instantaneous point source, $G(r,\theta,x,t|r',\theta',x',t')$.

When the Green's function $G(r,\theta,x,t|r',\theta',x')$ is known, the solution for the temperature caused by N concentrated sources at $(r_0, 0, pL)$ is given by

$$\begin{aligned}T(r,\theta,x,t)=&\frac{Q}{\rho c}\sum_{p=0}^{N-1}G(r,\theta,x,t|r_0,0,pL)\\&-\frac{\alpha}{h}\int_{-\pi}^{\pi}\int_{-\infty}^{\infty}f(\theta',x')\frac{\partial}{\partial r'}G(r,\theta,x,t|r_0,\theta',x')r_0\,\mathrm{d}\theta'\,\mathrm{d}x'\\&+\int_0^{r_0}\int_{-\pi}^{\pi}\int_{-\infty}^{\infty}F(r',\theta',x')G(r,\theta,x,t|r',\theta',x')r'\,\mathrm{d}r'\mathrm{d}\theta'\mathrm{d}x'\end{aligned} \tag{11}$$

and the temperature caused by N distributed sources at $(r_0,\theta',pL+x')$ is given by

$$\begin{aligned}T(r,\theta,x,t)=&\frac{r_0}{\rho c}\sum_{p=0}^{N-1}\int_{pL-l}^{pL+l}\int_{-v}^{v}Q''G(r,\theta,x,t|r_0,\theta',x')\,\mathrm{d}\theta'\,\mathrm{d}x'\\&-\frac{\alpha}{h}\int_{-\pi}^{\pi}\int_{-\infty}^{\infty}f(\theta',x')\frac{\partial}{\partial r'}G(r,\theta,x,t|r_0,\theta',x')r_0\,\mathrm{d}\theta'\,\mathrm{d}x'\\&+\int_0^{r_0}\int_{-\pi}^{\pi}\int_{-\infty}^{\infty}F(r',\theta',x')G(r,\theta,x,t|r',\theta',x')r'\,\mathrm{d}r'\,\mathrm{d}\theta'\,\mathrm{d}x'.\end{aligned} \tag{12}$$

In the above formulation, we have assumed that the solid is moving while the sources are stationary. Alternatively, we may consider the solid as stationary while the sources are moving helically over the cylinder. Now if we let the centroids of sources be seated at $(r_0, 0, pL)$, we can then expect that, as time goes to infinity, steady state will be reached. In practice, such a steady-state solution is of more importance. For this reason and others which will be seen later, we shall first consider the steady problem.

For steady state, the Green's function $G(r,\theta,x|r',\theta',x')$ is to satisfy the following equations

$$\frac{1}{\alpha}\left(\omega\frac{\partial G}{\partial\theta}+u\frac{\partial G}{\partial x}\right)-\nabla^2 G=\frac{1}{\alpha r}\delta(r-r')\delta(\theta-\theta')\delta(x-x') \tag{13}$$

for $0<r<r_0, 0\leqslant\theta\leqslant 2\pi, |x|<\infty$;

$$k\frac{\partial G}{\partial r}+hG=0 \quad \text{for} \quad r=r_0, \quad 0\leqslant\theta\leqslant 2\pi, \quad |x|<\infty \tag{14}$$

$$G=0 \quad \text{for} \quad 0<r<r_0, \quad 0\leqslant\theta\leqslant 2\pi, \quad |x|=\infty. \tag{15}$$

The temperature distributions for concentrated and distributed sources are given by equations in the same forms as (11) and (12) after the variable t is dropped.

*For simplicity, $\delta(t-0)$ has been deleted in the right hand side of the equation.

HELICALLY MOVING SOURCES—STEADY STATE

We seek the solution of (13) in the form

$$G(r,\theta,x|r',\theta',x') = \sum_{n=-\infty}^{\infty}\sum_{m=1}^{\infty} X_{nm}(x|x',r',\theta',\lambda_{nm})J_n(\lambda_{nm}r)e^{in\theta} \tag{16}$$

where J_n is the Bessel function of the first kind and order n. The expression (16) has already satisfied the finite condition at $r = 0$ and also satisfies condition (14) if λ_{nm} are the roots of

$$\lambda_{nm}J_n'(\lambda_{nm}r_0)+\frac{h}{k}J_n(\lambda_{nm}r_0) = 0. \tag{17}$$

Multiplying both sides of (13) by $rJ_n(\lambda_{nm}r)e^{-in\theta}\,dr\,d\theta$, integrating over $(0, r_0)$ and $(-\pi,\pi)$, and using the orthogonality of $J_n(\lambda_{nm}r)$ and $e^{in\theta}$, we obtain

$$X_{nm}''-\frac{u}{\alpha}X_{nm}'-(\lambda_{nm}^2+i\omega_n^2)X_{nm} = -\frac{J_n(\lambda_{nm}r')}{2\pi\alpha N_{nm}}\delta(x-x')e^{-in\theta'} \tag{18}$$

where N_{nm} is the norm given by

$$N_{nm} = \int_0^{r_0} rJ_n^2(\lambda_{nm}r)\,dr = \frac{r_0^2}{2\lambda_{nm}^2}\left[\left(\frac{h}{k}\right)^2+\lambda_{nm}^2-\frac{n^2}{r_0^2}\right]J_n^2(\lambda_{nm}r_0). \tag{19}$$

In view of (15), the boundary conditions on X_{nm} may be written as

$$X_{nm} = 0 \qquad \text{for} \qquad |x| = \infty. \tag{20}$$

Clearly, X_{nm} is the one-dimensional fundamental Green's function (i.e. Green's function in an infinite region) with strength $J_n(\lambda_{nm}r')e^{-in\theta'}/(2\pi\alpha N_{nm})$ and can be constructed from its basic properties [9] or by introducing $X_{nm} = \phi_{nm}e^{u(x-x')/2\alpha}$ to (18) so that

$$\phi_{nm}''-\left(\frac{u^2}{4\alpha^2}+\lambda_{nm}^2+i\omega_n^2\right)\phi_{nm} = -\frac{J_n(\lambda_{nm}r')}{2\pi\alpha N_{nm}}\delta(x-x')e^{-u(x-x')/2\alpha}e^{-in\theta'}.$$

The solution for ϕ_{nm} satisfying $\phi_{nm}(\pm\infty) = 0$ is

$$\phi_{nm}(x|x') = \frac{J_n(\lambda_{nm}r')\exp[-(u^2/4\alpha^2+\lambda_{nm}^2+i\omega_n^2)^{1/2}|x-x'|]}{4\pi\alpha N_{nm}[u^2/4\alpha^2+\lambda_{nm}^2+i\omega_n^2]^{1/2}}e^{-in\theta'}.$$

It follows that

$$X_{nm}(x|x') = \frac{J_n(\lambda_{nm}r')\exp\left[\frac{u}{2\alpha}(x-x')-\left(\frac{u^2}{4\alpha^2}+\lambda_{nm}^2+i\omega_n^2\right)^{1/2}|x-x'|\right]}{4\pi\alpha N_{nm}[u^2/4\alpha^2+\lambda_{nm}^2+i\omega_n^2]^{1/2}}e^{-in\theta'}. \tag{21}$$

Substituting (21) into (16) we obtain

$$G(r,\theta,x|r',\theta',x') = \frac{1}{2\pi r_0^2\alpha}\sum_{n=-\infty}^{\infty}\sum_{m=1}^{\infty}\frac{\lambda_{nm}^2J_n(\lambda_{nm}r)J_n(\lambda_{nm}r')e^{in(\theta-\theta')}}{[(h/k)^2+\lambda_{nm}^2-n^2/r_0^2]J_n^2(\lambda_{nm}r_0)}$$
$$\times\frac{1}{[u^2/4\alpha^2+\lambda_{nm}^2+i\omega_n^2]^{1/2}}\exp\left[\frac{u}{2\alpha}(x-x')-\left(\frac{u^2}{4\alpha^2}+\lambda_{nm}^2+i\omega_n^2\right)^{1/2}|x-x'|\right]. \tag{22}$$

After a little mathematical deduction (22) becomes

$$G(r,\theta,x|r',\theta',x') = \frac{\exp\left[\frac{u}{2\alpha}(x-x')\right]}{2\pi r_0^2\alpha}\sum_{m=1}^{\infty}\left[\frac{B_{om}(r,r')}{a_0}\exp(-a_0|x-x'|)+2\sum_{n=1}^{\infty}\frac{B_{nm}(r,r')\exp(-a_n|x-x'|)}{a_n^2+b_n^2}\right.$$
$$\left.\times\{a_n\cos[n(\theta-\theta')-b_n|x-x'|]+b_n\sin[n(\theta-\theta')-b_n|x-x'|]\}\right] \tag{23}$$

where

$$B_{nm}(r,r') = \frac{\lambda_{nm}^2 J_n(\lambda_{nm}r)J_n(\lambda_{nm}r')}{[(h/k)^2+\lambda_{nm}^2-n^2/r_0^2]J_n^2(\lambda_{nm}r_0)} \tag{24}$$

$$a_0 = [\lambda_{0m}^2+u^2/4\alpha^2]^{1/2}, \qquad b_0 = 0$$

$$a_n = \frac{1}{\sqrt{2}}\{[(\lambda_{nm}^2+u^2/4\alpha^2)^2+\omega_n^4]^{1/2}+(\lambda_{nm}^2+u^2/4\alpha^2)\}^{1/2} \tag{25}$$

$$b_n = \frac{1}{\sqrt{2}}\{[(\lambda_{nm}^2+u^2/4\alpha^2)^2+\omega_n^4]^{1/2}-(\lambda_{nm}^2+u^2/4\alpha^2)\}^{1/2}.$$

If $f = 0$ and the heat produced between each grinder (or cutter) and the cylinder is assumed to concentrate in a point, then the steady temperature distribution in the cylinder is obtained

$$T(r,\theta,x) = \frac{Q}{2\pi r_0^2 k}\sum_{p=0}^{N-1} e^{\frac{u}{2\alpha}(x-pL)}\sum_{m=1}^{\infty}\left\{\frac{B_{0m}(r,r_0)\,e^{-a_0|x-pL|}}{a_0}+2\sum_{n=1}^{\infty}\frac{B_{nm}(r,r_0)\,e^{-a_n|x-pL|}}{a_n^2+b_n^2}\right.$$
$$\left.\times\,[a_n\cos(n\theta-b_n|x-pL|)+b_n\sin(n\theta-b_n|x-pL|)]\right\}. \tag{26}$$

If the heat is generated uniformly from finite areas, the temperature distribution in steady state is

$$T(r,\theta,x) = \frac{Q''}{\rho c}\sum_{p=0}^{N-1}\int_{-v}^{v}\int_{pL-l}^{pL+l} G(r,\theta,x|r_0,\theta',x')\,dx'r_0\,d\theta'$$
$$= \frac{Q''}{2\pi r_0 k}\sum_{p=0}^{N-1}\int_{-v}^{v}\int_{pL-l}^{pL+l}\sum_{m=1}^{\infty}\left[\frac{B_{0m}(r,r_0)}{a_0}e^{\frac{u}{2\alpha}(x-x')-a_0|x-x'|}+2\sum_{n=1}^{\infty}\frac{B_{nm}(r,r_0)}{a_n^2+b_n^2}\right.$$
$$\left.\times\,e^{\frac{u}{2\alpha}(x-x')-a_n|x-x'|}\{a_n\cos[n(\theta-\theta')-b_n|x-x'|]+b_n\sin[n(\theta-\theta')-b_n|x-x'|]\}\right]dx'\,d\theta'. \tag{27}$$

Performing the integration with respect to θ', yields

$$T(r,\theta,x) = \frac{Q''}{\pi r_0 k}\sum_{p=0}^{N-1}\left\{v\sum_{m=1}^{\infty}\frac{B_{0m}(r,r_0)}{a_0^2}F_0(x,p)+2\sum_{n=1}^{\infty}\frac{\sin v}{n}\sum_{m=1}^{\infty}\frac{B_{nm}(r,r_0)}{a_n^2+b_n^2}F_n(x,\theta,p)\right\} \tag{28}$$

where

$$F_0(x,p) = a_0\int_{pL-l}^{pL+l} e^{\frac{u}{2\alpha}(x-x')-a_0|x-x'|}\,dx'$$
$$F_n(x,\theta,p) = F_{1n}(x,p)\cos n\theta+F_{2n}(x,p)\sin n\theta$$
$$F_{1n}(x,p) = \int_{pL-l}^{pL+l} e^{\frac{u}{2\alpha}(x-x')-a_n|x-x'|}[a_n\cos b_n|x-x'|-b_n\sin|x-x'|]\,dx' \tag{29}$$
$$F_{2n}(x,p) = \int_{pL-l}^{pL+l} e^{\frac{u}{2\alpha}(x-x')-a_n|x-x'|}[b_n\cos b_n|x-x'|+a_n\sin|x-x'|]\,dx'.$$

Due to the presence of $|x-x'|$ and three different zones $(-\infty, pL-l)$, $(pL-l, pL+l)$ and $(pL+l,\infty)$, the integration with respect to x' in (29) is quite complicated. By introducing Heavisides' step function to each zone, we obtain

$$F_0(x,p) = \frac{a_0}{a_0-\frac{u}{2\alpha}}\left[e^{-\left(a_0-\frac{u}{2\alpha}\right)H(x-pL-l)}-e^{-\left(a_0-\frac{u}{2\alpha}\right)H(x-pL+l)}\right]$$
$$+\frac{a_0}{a_0+\frac{u}{2\alpha}}\left[e^{-\left(a_0+\frac{u}{2\alpha}\right)H(-x+pL-l)}-e^{-\left(a_0+\frac{u}{2\alpha}\right)H(-x+pL+l)}\right] \tag{30}$$

$$F_{1n}(x,p)=\frac{e^{-\left(a_n-\frac{u}{2\alpha}\right)H(x-pL-l)}}{\left(a_n-\frac{u}{2\alpha}\right)^2+b_n^2}\left\{\left[a_n\left(a_n-\frac{u}{2\alpha}\right)-b_n^2\right]\cos b_nH(x-pL-l)-b_n\left[2a_n-\frac{u}{2\alpha}\right]\sin b_nH(x-pL-l)\right\}$$

$$-\frac{e^{-\left(a_n-\frac{u}{2\alpha}\right)H(x-pL+l)}}{\left(a_n-\frac{u}{2\alpha}\right)^2+b_n^2}\left\{\left[a_n\left(a_n-\frac{u}{2\alpha}\right)-b_n^2\right]\cos b_nH(x-pL+l)-b_n\left[2a_n-\frac{u}{2\alpha}\right]\sin b_nH(x-pL+l)\right\}$$

$$+\frac{e^{-\left(a_n+\frac{u}{2\alpha}\right)H(-x+pL-l)}}{\left(a_n+\frac{u}{2\alpha}\right)^2+b_n^2}\left\{\left[a_n\left(a_n+\frac{u}{2\alpha}\right)-b_n^2\right]\cos b_nH(-x+pL-l)-b_n\left[2a_n+\frac{u}{2\alpha}\right]\sin b_nH(-x+pL-l)\right\}$$

$$-\frac{e^{-\left(a_n+\frac{u}{2\alpha}\right)H(-x+pL+l)}}{\left(a_n+\frac{u}{2\alpha}\right)^2+b_n^2}\left\{\left[a_n\left(a_n+\frac{u}{2\alpha}\right)-b_n^2\right]\cos b_nH(-x+pL+l)-b_n\left[2a_n+\frac{u}{2\alpha}\right]\sin b_nH(-x+pL+l)\right\} \tag{31}$$

$$F_{2n}(x,p)=\frac{e^{-\left(a_n-\frac{u}{2\alpha}\right)H(x-pL-l)}}{\left(a_n-\frac{u}{2\alpha}\right)^2+b_n^2}\left\{\left[a_n\left(a_n-\frac{u}{2\alpha}\right)-b_n^2\right]\sin b_nH(x-pL-l)+b_n\left[2a_n-\frac{u}{2\alpha}\right]\cos b_nH(x-pL-l)\right\}$$

$$-\frac{e^{-\left(a_n-\frac{u}{2\alpha}\right)H(x-pL+l)}}{\left(a_n-\frac{u}{2\alpha}\right)^2+b_n^2}\left\{\left[a_n\left(a_n-\frac{u}{2\alpha}\right)-b_n^2\right]\sin b_nH(x-pL+l)+b_n\left[2a_n-\frac{u}{2\alpha}\right]\cos b_nH(x-pL+l)\right\}$$

$$+\frac{e^{-\left(a_n+\frac{u}{2\alpha}\right)H(-x+pL-l)}}{\left(a_n+\frac{u}{2\alpha}\right)^2+b_n^2}\left\{\left[a_n\left(a_n+\frac{u}{2\alpha}\right)-b_n^2\right]\sin b_nH(-x+pL-l)+b_n\left[2a_n+\frac{u}{2\alpha}\right]\cos b_nH(-x+pL-l)\right\}$$

$$-\frac{e^{-\left(a_n+\frac{u}{2\alpha}\right)H(-x+pL+l)}}{\left(a_n+\frac{u}{2\alpha}\right)^2+b_n^2}\left\{\left[a_n\left(a_n+\frac{u}{2\alpha}\right)-b_n^2\right]\sin b_nH(-x+pL+l)+b_n\left[2a_n+\frac{u}{2\alpha}\right]\cos b_nH(-x+pL+l)\right\} \tag{32}$$

where the function $H(x)$ is defined as

$$H(x)=\begin{cases}x & \text{for} \quad x>0\\ 0 & \text{for} \quad x\leqslant 0.\end{cases} \tag{33}$$

The detailed derivation of the above equations can be found in [10].

HELICALLY MOVING SOURCES—TRANSIENT AND UNSTEADY STATES

The Green's function for the unsteady problem, (7–10), may be written as

$$G(r,\theta,x,t|r',\theta',x')=g(r,\theta,x,t|r',\theta',x')+G(r,\theta,x|r',\theta',x') \tag{34}$$

where $G(r,\theta,x|r',\theta',x')$ is given by (23) and $g(r,\theta,x,t|r',\theta',x')$ satisfies

$$\frac{1}{\alpha}\left(\frac{\partial g}{\partial t}+u\frac{\partial g}{\partial x}+\omega\frac{\partial g}{\partial\theta}\right)-\nabla^2 g=0 \tag{35}$$

$$\left.\begin{aligned}
&g=-G(r,\theta,x|r',\theta',x') && 0<r<r_0, \quad 0\leqslant\theta\leqslant 2\pi, \quad |x|<\infty, \quad t=0\\
&k\frac{\partial g}{\partial r}+hg=0 && r=r_0, \quad 0\leqslant\theta\leqslant 2\pi, \quad |x|<\infty, \quad t>0\\
&g=0 && 0<r<r_0, \quad 0\leqslant\theta\leqslant 2\pi, \quad |x|=\infty, \quad t>0.
\end{aligned}\right\} \tag{36}$$

Clearly, (34) satisfies (7–10) and $g(r, \theta, x, t|r', \theta', x')$ can be readily recognized as the Green's functions associated with the transient problem. In view of the initial condition in (36) and $G(r, \theta, x|r', \theta', x')$ in (22) we seek the solution of (35) in the form

$$g = \sum_{n=-\infty}^{\infty} \sum_{m=1}^{\infty} A_{nm} \psi_{nm}(x, t|x') J_n(\lambda_{nm} r) e^{-in\theta - (\lambda_{nm}^2 + i\omega_n^2)\alpha t} \tag{37}$$

where

$$A_{nm} = \frac{-\lambda_{nm}^2 J_n(\lambda_{nm} r') e^{-in\theta'}}{2\pi r_0^2 \alpha [(h/k)^2 + \lambda_{nm}^2 - (n/r_0)^2] J_n^2(\lambda_{nm}^2 r_0) \left[\frac{u^2}{4\alpha^2} + \lambda_{nm}^2 + i\omega_n^2\right]^{1/2}} \tag{38}$$

and λ_{nm} are the roots of (17). Substituting (37) into (35) yields

$$\frac{\partial \psi_{nm}}{\partial t} + u \frac{\partial \psi_{nm}}{\partial x} - \alpha \frac{\partial^2 \psi_{nm}}{\partial x^2} = 0. \tag{39}$$

The boundary conditions on ψ_{nm} may be taken as

$$\begin{aligned} \psi_{nm} &= 0, \quad \text{for} \quad |x| < \infty, \quad t > 0 \\ \psi_{nm} &= \exp\left[\frac{u}{2\alpha}(x - x') - \left(\frac{u^2}{4\alpha^2} + \lambda_{nm}^2 + i\omega_n^2\right)^{1/2} |x - x'|\right], \quad \text{for} \quad |x| < \infty, \quad t = 0. \end{aligned} \tag{40}$$

The solution of (39) satisfying (40) can be readily written down as

$$\psi_{nm} = \int_{-\infty}^{\infty} \exp\left[\frac{u}{2\alpha}(x'' - x') - \left(\frac{u^2}{4\alpha^2} + \lambda_{nm}^2 + i\omega_n^2\right)^{1/2} |x'' - x'|\right] G(x, t|x'', 0)\, dx'' \tag{41}$$

where $G(x, t|x'')$ is the Green's function associated with (39)–(40) and is

$$G(x, t|x'') = \frac{1}{2\sqrt{(\pi\alpha t)}} \exp[-(x - x'' - ut)^2/(4\alpha t)]. \tag{42}$$

Substituting (42) into (41) and performing the integration yields

$$\psi_{nm}(x, t|x') = \tfrac{1}{2} e^{-(x-x'-ut)^2/4\alpha t} [e^{z_1^2} \operatorname{erfc}(z_1) + e^{z_2^2} \operatorname{erfc}(z_2)] \tag{43}$$

where erfc(z) is the complimentary error function of complex argument. z_1 and z_2 are defined by

$$\begin{aligned} z_1 &= \left(\frac{x - x'}{2\alpha t} + a_n \pm i b_n\right)\sqrt{(\alpha t)} = \xi_1 \pm i\eta \\ z_2 &= \left(-\frac{x - x'}{2\alpha t} + a_n \pm i b_n\right)\sqrt{(\alpha t)} = \xi_2 \pm i\eta \\ \xi_1 &= \frac{x - x'}{2\sqrt{(\alpha t)}} + a_n\sqrt{(\alpha t)}, \qquad \xi_2 = -\frac{x - x'}{2\sqrt{(\alpha t)}} + a_n\sqrt{(\alpha t)}, \qquad \eta = b_n\sqrt{(\alpha t)} \end{aligned} \tag{44}$$

where a_n and b_n have been defined in (25) and the minus sign is used when n = negative. Substituting (38) and (43) into (37) gives the Green's function for the transient state,

$$g(r, \theta, x, t|r', \theta', x') = \frac{-1}{4\pi r_0^2 \alpha} \sum_{n=-\infty}^{\infty} \sum_{m=1}^{\infty} \frac{B_{nm}(r, r') e^{in(\theta - \theta') - (\lambda_{nm}^2 + i\omega_n^2)\alpha t}}{\left[\frac{u^2}{4\alpha^2} + \lambda_{nm}^2 + i\omega_n^2\right]^{1/2}} \cdot [e^{z_1^2} \operatorname{erfc}(z_1) + e^{z_2^2} \operatorname{erfc}(z_2)] e^{-(x-x'-ut)^2/4\alpha t} \tag{45}$$

To reduce the complex functions in (45) to real functions, we separate the complex function $e^{z^2}\operatorname{erfc}(z)$ into real and imaginary parts by means of the following relations:

$$\begin{aligned} w(iz) &= e^{z^2} \operatorname{erfc}(z) \\ w(z) &= \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{e^{-\tau^2}}{z - \tau} d\tau = U(\xi, \eta) + iV(\xi, \eta) \end{aligned} \tag{46}$$

where $z = \xi + i\eta$ and $U(\xi,\eta)$ and $V(\xi,\eta)$ have been tabulated [11]. From these relations, the following corrolaries which are needed here can be obtained:

$$\begin{aligned} w(iz) &= U(\eta,\xi) - iV(\eta,\xi) \\ w(i\bar{z}) &= U(\eta,\xi) + iV(\eta,\xi) \end{aligned} \tag{46'}$$

where $\bar{z} = \xi - i\eta$. Applying (46) and (46′) to (45) we obtain the Green's function for the transient problem

$$\begin{aligned} g(r,\theta,x,t|r',\theta',x') = \frac{-1}{2\pi r_0^2\alpha}\sum_{m=1}^{\infty}\Bigg[\frac{B_{0m}(r,r')}{2a_0}\Bigg\{e^{\left(a_0+\frac{u}{2\alpha}\right)(x-x')}\operatorname{erfc}\left(a_0\sqrt{(\alpha t)}+\frac{x-x'}{2\sqrt{(\alpha t)}}\right) \\ + e^{-\left(a_0-\frac{u}{2\alpha}\right)(x-x')}\operatorname{erfc}\left(a_0\sqrt{(\alpha t)}-\frac{x-x'}{2\sqrt{(\alpha t)}}\right)\Bigg\} + e^{-(x-ut)^2/4\alpha t}\sum_{n=1}^{\infty}\frac{B_{nm}(r,r')}{a_n^2+b_n^2} \\ \times e^{-\lambda_{nm}^2\alpha t}\{\zeta_1\cos[n(\theta-\theta')-\omega_n^2\alpha t]+\zeta_2\sin[n(\theta-\theta')-\omega_n^2\alpha t]\}\Bigg] \end{aligned} \tag{47}$$

where

$$\begin{aligned} \zeta_1 &= a_n[U(\eta,\xi_1)+U(\eta,\xi_2)]+b_n[V(\eta,\xi_1)+V(\eta,\xi_2)] \\ \zeta_2 &= b_n[U(\eta,\xi_1)+U(\eta,\xi_2)]+a_n[V(\eta,\xi_1)+V(\eta,\xi_2)] \end{aligned} \tag{48}$$

and ξ_1, ξ_2 and η have been defined in (44).

If each of the N point sources generates heat at the rate of Q, the temperature history in the transient state with $F = f = 0$ is obtained by substituting g in (47) for G into (11) and replacing x' by pL and θ' by zero.

$$\begin{aligned} T(r,\theta,x,t) &= \frac{Q}{\rho c}\sum_{p=0}^{N-1} g(r,\theta,x,t|r_0,0,pL) \\ &= -\frac{Q}{2\pi r_0^2 k}\sum_{p=0}^{N-1} e^{\frac{u}{2\alpha}(x-pL)}\sum_{m=1}^{\infty}\Bigg\{\frac{B_{0m}(r,r_0)}{2a_0}\Bigg[e^{a_0(x-pL)}\operatorname{erfc}\left(a_0\sqrt{(\alpha t)}+\frac{x-pL}{2\sqrt{(\alpha t)}}\right) \\ &\quad + e^{-a_0(x-pL)}\operatorname{erfc}\left(a_0\sqrt{(\alpha t)}-\frac{x-pL}{2\sqrt{(\alpha t)}}\right)\Bigg] + e^{-(x-pL-ut)^2/4\alpha t}\sum_{n=1}^{\infty}\frac{B_{nm}(r,r_0)}{a_n^2+b_n^2} \\ &\quad \times e^{-\lambda_{nm}^2\alpha t}[\zeta_1\cos(n\theta-\omega_n^2\alpha t)+\zeta_2\sin(n\theta-\omega_n^2\alpha t)]\Bigg\}. \end{aligned} \tag{49}$$

Similarly, for N distributed sources each generating heat at the rate of Q'' per unit area, the temperature history can be obtained from (12); the integration cannot be expressed in tabulated functions but it can be obtained numerically without difficulty.

MOVING RING SOURCES

If the rotational velocity ω of the cylinder is very large in comparison with the translational velocity, u, the problem may be approximated as one of moving ring sources. Then the temperature will be independent of the angular position θ. The Green's functions for steady, transient and unsteady states can be similarly obtained by the direct solution of the appropriate differential equations [10], or by specializing the results of (23) and (45) to the present cases. Setting $\theta = \omega = 0$ in (23) and (45), integrating with respect to θ' from $-\pi$ to π and dividing by 2π, we obtain the Green's functions in steady and transient states,

$$G(r,x|r',x') = \frac{e^{\frac{u}{2\alpha}(x-x')}}{r_0\alpha}\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0(\lambda_m r)J_0(\lambda_m r')e^{-a_0|x-x'|}}{a_0 J_0^2(\lambda_m r_0)[\lambda_m^2+(h/k)^2]} \tag{50}$$

$$\begin{aligned} g(r,x,t|r',x') = \frac{-1}{2r_0\alpha}\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0(\lambda_m r)J_0(\lambda_m r')e^{\frac{u}{2\alpha}(x-x')+\lambda_m^2\alpha t}}{a_0 J_0^2(\lambda_m r_0)[\lambda_m^2+(h/k)^2]} \\ \times\left[e^{a_0(x-x')}\operatorname{erfc}\left(a_0\sqrt{(\alpha t)}+\frac{x-x'}{2\sqrt{(\alpha t)}}\right)+e^{-a_0(x-x')}\operatorname{erfc}\left(a_0\sqrt{(\alpha t)}-\frac{x-x'}{2\sqrt{(\alpha t)}}\right)\right] \end{aligned} \tag{51}$$

where a_0 has been defined in (25) and λ_m are the roots of (17) with $n = 0$. It is to be noted that (50) and (51) represent the first series in (23) and (47), respectively, i.e. for $n = 0$. Thus, the series for $n \neq 0$ represent the correction terms for the tangential heat flow.

If heat is generated at the rate of Q' per unit length of the ring at $(r_0, 0)$, the steady and transient temperature distributions in the cylinder due to a single ring can be readily written down by multiplying $Q'/\rho c$ to the right hand side of (50) and (51) and setting $r' = r_0$ and $x' = 0$. The steady solution has been reported in [5].

If there are N ring sources apart at a distance L, the temperature distributions in steady and transient states are

$$T(r,x) = \frac{Q'}{r_0 k} \sum_{p=0}^{N-1} \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m r)\, e^{\frac{u}{2\alpha}(x-pL) - a_0|x-pL|}}{a_0 J_0(\lambda_m r_0)[\lambda_m^2 + (h/k)^2]} \tag{52}$$

$$T(r,x,t) = -\frac{Q'}{2r_0 k} \sum_{p=0}^{N-1} e^{\frac{u}{2\alpha}(x-pL)} \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m r)}{a_0 J_0(\lambda_m r_0)[\lambda_m^2 + (h/k)^2]}$$
$$\times \left[e^{a_0(x-pL)} \operatorname{erfc}\left(a_0 \sqrt{(\alpha t)} + \frac{x-pL}{2\sqrt{(\alpha t)}} \right) + e^{-a_0(x-pL)} \operatorname{erfc}\left(a_0 \sqrt{(\alpha t)} - \frac{x-pL}{2\sqrt{(\alpha t)}} \right) \right]. \tag{53}$$

If each of the ring sources has width of $2l$, the temperature distribution in steady state is

$$T(r,x) = \frac{Q''}{2\pi r_0^2 k} \sum_{p=0}^{N-1} \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m r)}{a_0 J_0(\lambda_m r_0)[\lambda_m^2 + (h/k)^2]} . 2\pi r_0 \int_{pL-l}^{pL+l} e^{\frac{u}{2\alpha}(x-x') - a_0|x-x'|} dx' \tag{54}$$

$$= \frac{Q''}{r_0 k} \sum_{p=0}^{N-1} \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0(\lambda_m r) F_0(x,p)}{a_0^2 J_0(\lambda_m r_0)[\lambda_m^2 + (h/k)^2]} \tag{55}$$

where F_0 is given in (29).

GENERATRIX LINE SOURCE

Consider a generatric line source at (r', θ') rotating about the cylinder axis at a constant angular velocity, ω. To find the Green's functions for the transient and steady problems, we may specialize (23) and (47) to this case by setting $u = 0$ and $x = 0$, and integrating with respect to x over $(-\infty, \infty)$. The results are:

$$G(r,\theta|r',\theta') = \frac{1}{\pi r_0^2 \alpha} \sum_{m=1}^{\infty} \left\{ \frac{B_{0m}(r,r')}{\lambda_{0m}^2} + 2 \sum_{n=1}^{\infty} \frac{B_{nm}(r,r')}{(\lambda_{nm}^4 + \omega_n^4)} . [\lambda_{nm}^2 \cos n(\theta-\theta') + \omega_n^2 \sin n(\theta-\theta')] \right\} \tag{56}$$

$$g(r,\theta,t|r',\theta') = \frac{-1}{\pi r_0^2 \alpha} \sum_{m=1}^{\infty} \left[\frac{B_{0m}(r,r')}{\lambda_{0m}^2} e^{-\lambda_{0m}^2 \alpha t} + 2 \sum_{n=1}^{\infty} \frac{B_{nm}(r,r')}{(\lambda_{nm}^4 + \omega_n^4)} \right.$$
$$\left. \times \{ \lambda_{nm}^2 \cos[n(\theta-\theta') - \omega_n^2 \alpha t] + \omega_n^2 \sin[n(\theta-\theta') - \omega_n^2 \alpha t] \} \right]. \tag{57}$$

These results can also be obtained from the Green's function associated with heat equation [10]. However, $G(r,\theta|r',\theta')$ can be expressed in the form of a single series, were it determined directly from its governing equations (13) and (14) after $\delta(x-x')$, $|x| < \infty$, and all derivatives with respect to x are deleted. We now write

$$G(r,\theta|r',\theta') = \sum_{n=-\infty}^{\infty} R_n(r|r';n) e^{in\theta} \tag{58}$$

Multiplying both sides of the differential equation by $e^{-in\theta}$, integrating with respect to θ over $(-\pi, \pi)$ and making use of the orthogonality of $e^{in\theta}$, we can obtain

$$R_n'' + \frac{1}{r} R_n' - \left(\frac{n^2}{r^2} + i\omega_n^2 \right) R_n = \frac{e^{in\theta'}}{\alpha r} \delta(r-r'). \tag{59}$$

The solution of (59) satisfying the boundary condition

$$kR_n' + hR_n = 0 \qquad \text{for} \qquad r = r_0 \tag{60}$$

can be easily found as

$$R_n(r|r') = -\frac{u_1(r)u_2(r')}{p(r')W(u_1,u_2)} \qquad \text{and} \qquad -\frac{u_1(r')u_2(r)}{p(r')W(u_1,u_2)} \tag{61}$$

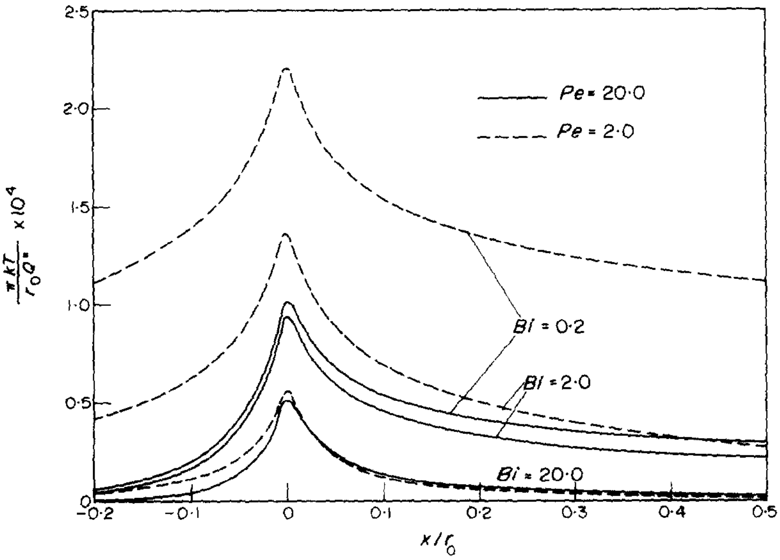

FIG. 2. Surface temperature due to single distributed heat source at $(r_0, 0, 0)$ vs axial position for $\theta = 0$ and $\omega = 10\pi$ rad/s.

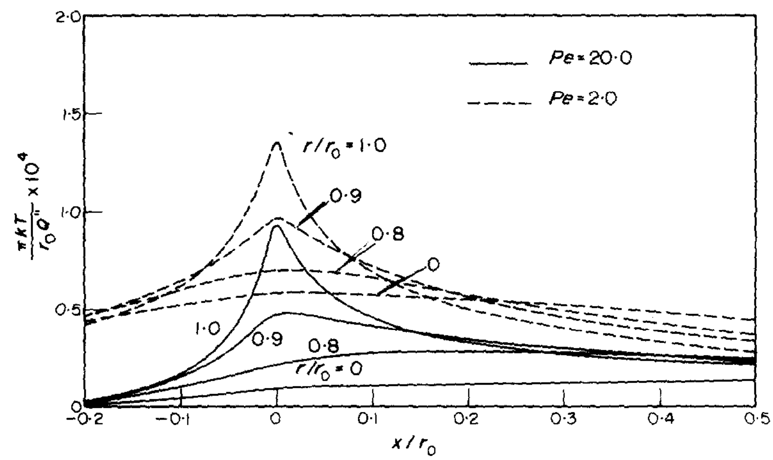

FIG. 3. Temperature due to a single distributed heat source at $(r, 0, 0)$ vs axial and radial positions for $\theta = 0$, $Bi = 2{\cdot}0$ and $\omega = 10\pi$ rad/s.

for $r < r'$ and $r > r'$, respectively, with $p(r') = r'$ and

$$\begin{aligned} u_1 &= I_n(\sqrt{i}\,.\,\omega_n r) \\ u_2 &= \{[hK_n(\sqrt{i}\,.\,\omega_n r_0) + k\sqrt{i}\,.\,\omega_n K_n'(\sqrt{i}\,.\,\omega_n r_0)]I_n(\sqrt{i}\,.\,\omega_n r) \\ &\quad - [hI_n(\sqrt{i}\,.\,\omega_n r_0) + k\sqrt{i}\,.\,\omega_n I_n'(\sqrt{i}\,.\,\omega_n r_0)]K_n(\sqrt{i}\,.\,\omega_n r)\} \qquad (62) \\ W(u_1, u_2) &= -\frac{1}{r'}[hI_n(\sqrt{i}\,.\,\omega_n r_0) + k\sqrt{i}\,.\,\omega_n I_n'(\sqrt{i}\,.\,\omega_n r_0)] \end{aligned}$$

where I_n and K_n are the modified Bessel functions of the first and second kinds, respectively. Rewriting the exponential and Bessel functions in complex forms and putting $r' = r_0$, (58) becomes

$$G(r,\theta|r_0,\theta') = \frac{k/h}{2\pi r_0\alpha} + \frac{k}{\pi\alpha}\sum_{n=1}^{\infty} \frac{M_1\left\{r_0\omega_n M_2\cos[n(\theta-\theta')+\phi_1-\phi_2] + \frac{hr_0}{k}M_3\cos[n(\theta-\theta')+\phi_1-\phi_3]\right\}}{r_0^2\omega_n^2 M_2^2 + \frac{h^2r_0^2}{k}M_3^2 + 2r_0\omega_n M_2\frac{hr_0}{k}M_3\cos(\phi_3-\phi_2)} \qquad (63)$$

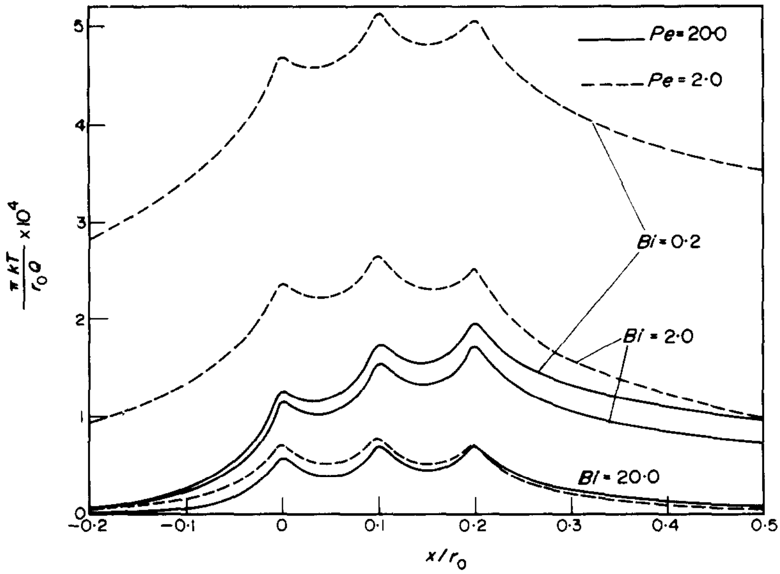

FIG. 4. Surface temperature due to triple distributed heat sources at $(r_0, 0, 0)$, $(r_0, 0, 0{\cdot}1)$, and $(r_0, 0, 0{\cdot}2)$ vs axial positions for $\theta = 0$ and $\omega = 10\pi$ rad/s.

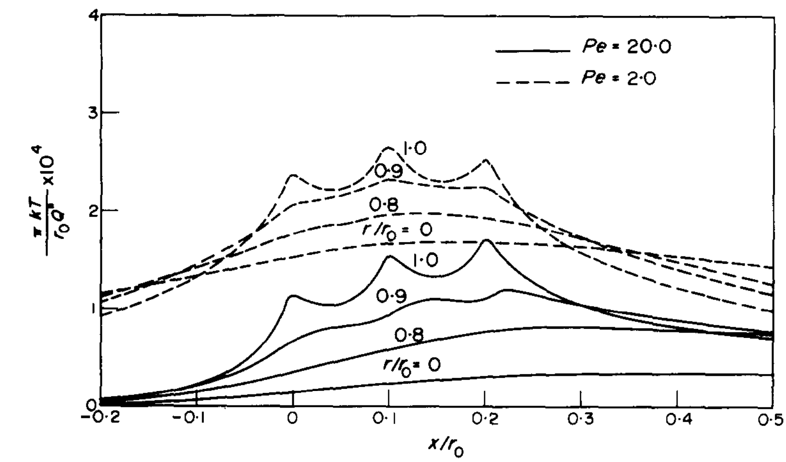

FIG. 5. Temperature due to triple distributed heat sources at $(r_0, 0, 0)$, $(r_0, 0, 0{\cdot}1)$, and $(r_0, 0, 0{\cdot}2)$ vs axial and radial positions for $\theta = 0$, $Bi = 2{\cdot}0$ and $\omega = 10\pi$ rad/s.

where

$$M_1 = [ber_n^2(\omega_n r) + bei_n^2(\omega_n r)]^{1/2}, \qquad M_2 = [ber_n'^2(\omega_n r_0) + bei_n'^2(\omega_n r_0)]$$

$$M_3 = [ber_n^2(\omega_n r_0) + bei_n^2(\omega_n r_0)]^{1/2}, \qquad \phi_1 = \tan^{-1}\frac{bei_n(\omega_n r)}{ber_n(\omega_n r)} \tag{64}$$

$$\phi_2 = \tan^{-1}\frac{bei_n'(\omega_n r_0)}{ber_n'(\omega_n r_0)}, \qquad \phi_3 = \tan^{-1}\frac{bei_n(\omega_n r_0)}{ber_n(\omega_n r_0)}.$$

and *ber* and *bei* stand for Bessel-real and Bessel-imaginary, respectively. If we replace θ' by ωt and multiply the right hand side of (63) by $Q'/\rho c$, we obtain the temperature distribution in periodic state due to a line source. If we replace θ' by $\omega t + \theta''$ and integrate with respect to θ'' over $(-v, v)$, we obtain the periodic solution for a band source which has been reported in [8].

RESULTS FOR STEADY STATE AND DISCUSSIONS

Equations (26) and (28) were calculated for $r_0 = 0{\cdot}5$ ft, $\alpha = 0{\cdot}172$ ft^2/h, $l = 0{\cdot}005 r_0$, $v = 0{\cdot}01$ rad, and a wide range of values of angular velocity, ω, Peclet number ($Pe = 2r_0 u/\alpha$) and Biot number ($Bi = 2r_0 h/k$). It was found that in the practical range of machining processes, $Pe = 2 \rightarrow 20$ and $\omega = 0{\cdot}1\pi \rightarrow 10\pi$ rad/s, the first series of (26) and (28) yield good results everywhere in the cylinder except in the proximity of a source. In other words, the temperature dependence of θ can be neglected and consequently the ring source approximation gives sufficiently accurate results for the practical range of machine operation.

Some of the calculated results are shown in Figs. 2–5. In Fig. 2 are shown the temperature distributions along the generatrix $(\theta = 0, r = r_0)$ due to a single distributed source at $(r' = r_0, -v < \theta' < v, x' = 0)$, for $Bi = 0{\cdot}2, 2{\cdot}0$. Temperature distributions along generatrices $(\theta \neq 0, r = r_0)$ are almost the same as those shown in Fig. 2 except near $x = 0$, as mentioned earlier. Figure 3 shows the temperature distribution in the cylinder due to a single distributed source at $(r' = r_0, -v < \theta' < v, x' = 0)$ for $\theta = 0$, $\omega = 10\pi$ rad/s and various values of *Pe* and *Bi*. In Figs. 4 and 5 are shown the temperature distributions on the surface and in the interior of the cylinder, respectively, under the influence of triple distributed sources.

From Figs. 2 and 4 it is seen that for large values of *Bi* (i.e. large rate of surface cooling), the variation of *Pe* (i.e. the variation of translational velocity) has little influence to the surface temperature, whereas the influence of *Pe* is quite large for small values of *Bi*. This discussion holds also to a lesser degree if we interchange *Bi* and *Pe* in the above statement. Figures 3 and 5 show that, for a given material with a given value of the convectance, the temperature in the cylinder can be higher than that on the surface at points away from the source or sources. The asymmetry of temperature curves in front of and behind the heat sources is more pronounced in the case of multiple sources than that of a single source. For small values of *Pe* or large values of *Bi* the maximum temperature occurs at the position of the middle source, whereas for large values of *Pe* or smaller values of *Bi* the maximum temperature takes place at the position of the right-hand source.

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UN SYSTEME DE SOURCES THERMIQUES CONCENTREES OU DISTRIBUEES SUR UN CYLINDRE EN MOUVEMENT DE TRANSLATION ET DE ROTATION

Résumé—Ce mémoire présente l'analyse de la conduction thermique dans un cylindre en mouvement de translation et de rotation, avec des sources de chaleur concentrées ou distribuées sur la surface. On considère les différents régimes: permanent, transitoire et variable. Physiquement, ce problème représente l'usinage d'un cylindre par un ensemble d'outils. Deux cas simples sont considérés: un système annulaire de source mobiles et une source mobile en forme de bande le long d'une génératrice.

EIN SYSTEM DISKRETER UND VERTEILTER WÄRMEQUELLEN AUF EINEM TRANSLATORISCH UND ROTIEREND BEWEGTEN ZYLINDER

Zusammenfassung—Diese Abhandlung stellt eine Analyse der Wärmeleitung in einem translatorisch und rotierend bewegten Zylinder unter dem Einfluß eines Systems von diskreten und verteilten Wärmequellen über die Zylinderoberfläche dar. Stationäre und instationäre Zustände sowie Übergangsphasen werden behandelt. Physikalisch stellt das Problem die Bearbeitung eines Zylinders mit einem Werkzeugsatz dar. Zwei einfachere Fälle wurden ebenfalls untersucht: ein System von bewegten Ringquellen und eine bewegte Linienquelle.

СИСТЕМА КОНЦЕНТРИРОВАННЫХ И РАСПРЕДЕЛЕННЫХ ТЕПЛОВЫХ ИСТОЧНИКОВ НА ЦИЛИНДРЕ, НАХОДЯЩЕМСЯ В ПОСТУПАТЕЛЬНОМ И ВРАЩАТЕЛЬНОМ ДВИЖЕНИИ

Аннотация — Представлен анализ теплопроводности в цилиндре, находящемся в поступательном и вращательном движении, при наличии системы концентрированных и распределенных источников на поверхности цилиндра. Рассматриваются стационарное, переходное и нестационарное состояния. Физически эта проблема имеет место при обработке цилиндра набором инструментов. Исследуются два простых случая: система движущихся кольцевых источников и источников, движущихся по образующей поверхности.